# Vortex matter, effective magnetic charges, and generalizations of the dipolar superfluidity concept in layered systems

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In the first part of this paper, we discuss electrodynamics of an excitonic condensate in a bilayer. We show that under certain conditions, the system has a dominant energy scale and is described by the effective electrodynamics with "planar magnetic charges." In the second part of the paper, we point out that a vortex liquid state in bilayer superconductors also possesses dipolar superfluid modes and establish equivalence mapping between this state and a dipolar excitonic condensate. We point out that a vortex liquid state in a $N$-layer superconductor possesses multiple topologically coupled dipolar superfluid modes and therefore represents a generalization of the dipolar superfluidity concept.

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## I. INTRODUCTION

The progress in semiconductor technology has made it feasible to produce bilayers where the interparticle distance is larger than the separation of the layers and which are expected to have interlayer excitonic states[1–5] (as shown in Fig. 1) with a significant lifetime. Exciton is a boson and can undergo Bose-Einstein condensation.[6] Some signatures of Bose-Einstein condensate of interlayer excitons were reported.[7] A renewed theoretical interest in these systems[1–5] is focused on identification of possible unique properties of such condensates which can set them apart from other families of quantum fluids. In Ref. 4, it was discussed in great detail that the spatial separation of the positive and negative charges, in an exciton in a bilayer, makes the phase $\theta$ of the condensate transform as $\nabla\theta \to \nabla\theta - e\mathbf{A}(\mathbf{r}_+) + e\mathbf{A}_-(\mathbf{r}_-)$. The dipolar excitonic condensate (DEC) therefore features a coupling to a difference of vector potential values at positions in different layers $\mathbf{r}_+$ and $\mathbf{r}_-$.[1,2,4] In the case of a small layer separation $d$, one finds $\mathbf{A}(\mathbf{r}_+) - \mathbf{A}_-(\mathbf{r}_-) \approx d\partial_z \mathbf{A}(\mathbf{r})$. Therefore, a static in-plane magnetic field $\mathbf{H}^{ext}$ produces excitonic currents and the system is described by the following free energy density:[1,2,4]

$$\mathcal{F} = \frac{\rho}{2}(\nabla\theta + ed[\hat{R}\mathbf{H}^{ext}])^2, \tag{1}$$

where the operator $\hat{R}$ rotates a vector 90° counterclockwise, $\hat{R}(a,b,c) = (-b,a,c)$, $\rho$ is the phase stiffness, $e$ is the electric charge, and $d$ is the separation of the layers. A particularly interesting aspect of this observation is that although an exciton is, by definition, an electrically neutral object, it has the dipolar coupling to a gauge field and its effective low-energy model has a symmetry different from the so-called "global" $U(1)$ symmetry which one finds in ordinary neutral superfluids. It is also different from the "gauged" $U(1)$ symmetry which one finds in superconductors.

## II. EFFECTIVE MODEL AND VORTICES IN DIPOLAR CONDENSATE

To describe topological defects, we should include dynamics of the gauge field in model (1). Consider a system of two thin parallel layers with positive and negative charges bound in pairs (as shown in Fig. 1). The magnetic field obeys the Maxwell equations outside the layers. Here,

$$\nabla \times \mathbf{B} = \mathbf{J} = e[\delta(z_-) - \delta(z_+)] \times \left\{ \frac{i}{2}(\psi\nabla\psi^* - \psi^*\nabla\psi) - e|\psi|^2[\mathbf{A}(\mathbf{r}_+) - \mathbf{A}(\mathbf{r}_-)] \right\}, \tag{2}$$

where $\mathbf{J}$ is the electric current, $\psi$ is the DEC order parameter, $z_{(+,-)}$ are the $z$ axis positions of the upper and lower planes, and $\mathbf{r}_{(+,-)} = (x, y, z_{(+,-)})$. Topological defects in this system correspond to the situation when the phase $\theta$ of the order parameter $\psi$ changes by $2\pi n$. A configuration of accompanying magnetic field should be determined by minimization of the energy taking into account (i) the kinetic energy of currents in two planes, (ii) the potential energy of the corresponding Ginzburg-Landau functional, and (iii) the energy of the *three-dimensional* magnetic field configuration. This problem is nonlocal and the field-inducing current itself depends on the gauge field, i.e., should be determined in a self-consistent way. However, we show here that there is a regime when the system is accurately described by an unusual, on the other hand, tractable effective model. That is, let us consider the situation where the separation of the layers $d$ is small compared to the system size and $\mathbf{A}(\mathbf{r}_+)$

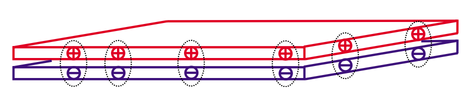

FIG. 1. (Color online) A schematic picture of DEC. An exciton forms as a result of the pairing of an electron in an electron-rich layer and a hole in a hole-rich layer.

 

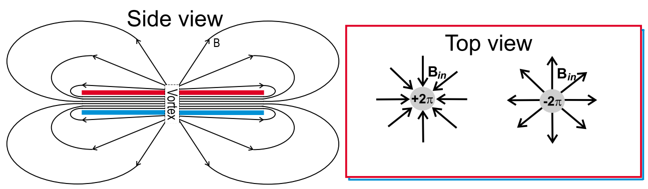


FIG. 2. (Color online) A schematic picture of a magnetic field configuration in DEC with vortices. Side view: the out-of-plane magnetic field $\mathbf{B}$ carries the same flux as the interplane field $\mathbf{B}_{in}$; however, it has the freedom to minimize energy by spreading above and below the top and bottom layers. Top view: the in-plane magnetic field $\mathbf{B}_{in}$ configuration for vortices with phase windings $\Delta\theta = \pm 2\pi$. As explained in the text, for a large system, one can neglect the energy of the out-of-plane field compared to the energy of the field in the dipolar bilayer $\mathbf{B}_{in}$.

$-\mathbf{A}_{-}(\mathbf{r}_{-}) \approx d\partial_z \mathbf{A}(\mathbf{r})$. Then, in the hydrodynamic limit, the effective model is

$$\mathcal{F}_{eff} = \frac{1}{2}\delta(z)\rho(\nabla\theta + ed[\hat{R}\mathbf{B}_{in}(\mathbf{r})])^2 + \frac{1}{2}\mathbf{B}^2(\mathbf{r}), \tag{3}$$

where $\mathbf{B}_{in}$ is the field in the dipolar layer. Consider a vortex with $\Delta\theta = 2\pi$. Then, $\nabla\theta = \frac{1}{r}\mathbf{e}_\theta$ which produces a logarithmic divergence of the energy in a neutral system. However, Eq. (3) suggests that for a given phase winding, the system can minimize its energy by generating a certain configuration of $\mathbf{B}_{in}$. Note that the second term in Eq. (3) depends quadratically on $\mathbf{B}$ and does not allow a configuration of $\mathbf{B}_{in}$ which would completely compensate the phase gradient in the first term. This is in contrast to a vortex in a superconducting film where the gauge field compensates the phase gradient at a certain distance making the vortex a finite-energy object. A configuration of the interplane field $\mathbf{B}_{in}$, which would partially compensate the divergence caused by $\nabla\theta$, should satisfy the condition,

$$ed(\hat{R}\mathbf{B}_{in}) \propto \frac{1}{r}\mathbf{e}_\theta \quad (r \gg r_{core}). \tag{4}$$

This implies the following self-induced interplane field:

$$\mathbf{B}_{in} = \alpha\frac{1}{ed}\frac{\mathbf{r}}{r^2} \quad [\alpha < 0, \mathbf{r} = (x,y,0)]. \tag{5}$$

At a first glance, Eq. (5) appears to violate the condition that the magnetic field should be divergenceless. This problem is resolved by including in the picture the "out-of-plane" magnetic fields. These, as schematically shown in Fig. 2, can restore the $\nabla\cdot\mathbf{B}=0$ constraint in three-dimensional physical space while permitting the in-plane field to have the form required by Eq. (5) with a natural cutoff close to the vortex core. From the $\nabla\cdot\mathbf{B}=0$ condition, it follows that both the interlayer $\mathbf{B}_{in}$ field and the out-of-plane field carry the same flux. However, for a given magnetic flux, the out-of-plane field has the freedom to spread in the positive and negative $z$ axis directions. Since the magnetic flux is $\int\mathbf{B}\cdot d\mathbf{S}$, while the magnetic field energy is $(1/2)\int_{\mathbf{r}}\mathbf{B}^2$, the out-of-plane field will have a finite value of the integral $(1/2)\int_{\mathbf{r},z\notin[z_-\ldots z_+]}\mathbf{B}^2$ over the entire three-dimensional space excluding the bilayer space. On the other hand, the magnetic field inside the bilayer behaves as $|\mathbf{B}_{in}| \propto 1/r$ and thus has logarithmically divergent energy $(1/2)\int_{\mathbf{r},z\in[z_-\ldots z_+]}\mathbf{B}^2$. Therefore, from the condition $\nabla\cdot\mathbf{B}=0$ and geometry of the problem, it follows that the energy of out-of-plane magnetic fields is negligible compared to the energy of magnetic field in the dipolar bilayer for a sample with $d$ much smaller than the system size.

Thus, we have identified the regime where the dynamics of the magnetic field is dominated by its most energetically costly (weakly divergent) interlayer part, $\mathbf{B}_{in}$, which leads to an interesting two-dimensional effective model where the magnetic field $\mathbf{B}_{in}$ is *not* subject to the constraint that $\nabla\cdot\mathbf{B}_{in}=0$,

$$\mathcal{F}_{eff}(x,y) = \frac{\rho}{2}(\nabla\theta + ed[\hat{R}\mathbf{B}_{in}(x,y)])^2 + \frac{d}{2}\mathbf{B}_{in}^2(x,y). \tag{6}$$

Let us consider a vortex with a phase winding $\Delta\theta = 2\pi n$ in model (6). The coefficient $\alpha_{min}$ for the ansatz [Eq. (5)] which minimizes the spatially integrated free energy density [Eq. (6)] for vortex with $\Delta\theta=2\pi n$ is $\alpha_{min}^{\Delta\theta=2\pi n} = -n\rho[\rho+(e^2d)^{-1}]^{-1}$. Thus, the vortex has the following configuration of the magnetic field:

$$\mathbf{B}_{in}^{\Delta\theta=2\pi n} = -n\frac{\rho}{\rho+(e^2d)^{-1}}\frac{1}{ed}\frac{\mathbf{r}}{r^2}, \tag{7}$$

where $\mathbf{r}=(x,y,0)$ (shown on Fig. 2). Therefore, these defects emit a quantized *radial* magnetic flux $\Phi = |\mathbf{B}(\mathbf{r})|2\pi r d$. The quantization condition is

$$\Phi^{\Delta\theta=2\pi n} = -\frac{2\pi}{e}\frac{\rho}{\rho+(e^2d)^{-1}}n. \tag{8}$$

Thus a "dipolar flux quantum" is

$$\Phi_d^0 = \frac{2\pi}{e}\frac{\rho e^2 d}{\rho e^2 d + 1} \leq \Phi_0, \tag{9}$$

where $\Phi_0 = 2\pi/e$ is the standard magnetic flux quantum. Note that this quantization has a different origin and character than that in multicomponent superconductors.[8] Observe that in the limit of zero dipolar coupling or zero $d$, the magnetic flux tends to zero because the system reduces in that limit to an electrically neutral condensate. Though, strictly speaking, we cannot take the limit $d\to\infty$ in this model, we can observe that with increasing $d$, the dipolar flux quantum saturates to the standard flux quantum. The same is achieved in the limit of large $\rho$. Though such limits are not expected to arise in DEC, it shows that the model captures important physical circumstance that $\Phi_d^0$ has a bound and this is more relevant for the system considered in the second part of the paper.

The existence of a radial quantized magnetic flux, which energy is much higher than that of the out-of-plane field, means that in a way, these topological excitations play a role of positive and negative "magnetic charges" in the effective planar electrodynamics of model (6). The logarithmically divergent part of the energy of such a vortex placed in the center of a circle-shaped system of a radius $R$ is

$$E \approx \frac{\pi \rho n^2}{1+\rho e^2 d} \ln \frac{R}{r_{core}}, \tag{10}$$

where $r_{core}$ is the vortex core size. Observe that in the limit $d \to 0$, the logarithmically divergent part becomes the same as that of a vortex of a neutral condensate with density $\rho$. Again, even though, strictly speaking, one cannot take $d \to \infty$ limit in this model, nonetheless we see that in that limit, the logarithmically divergent part of the vortex energy vanishes.

Even though the energy [Eq. (10)] is logarithmically divergent, these vortices can, in principle, be induced in a DEC by an external in-plane magnetic field because their magnetic field also has divergent energy. This provides a possibility to obtain a negative contribution to the Gibbs energy from $\int_{\mathbf{r}} \mathbf{B} \cdot \mathbf{H}$. For example, in principle, an experimentally realizable configuration of induction elements can produce the following external in-plane field $\mathbf{H} \approx \gamma \frac{\mathbf{r}}{r^2}$ [$\mathbf{r}=(x,y,0); r > r_0$], where $r_0$ is some cutoff length which is specific to the geometry of the induction elements. Then, for a vortex with $\Delta\theta = 2\pi$, the integral $\int_{\mathbf{r}} \mathbf{B} \cdot \mathbf{H}$ provides a negative logarithmically divergent contribution to the Gibbs energy $G = \int_{\mathbf{r}} (\mathcal{E} - \mathbf{B} \cdot \mathbf{H})$. There is a critical value of the coefficient $\gamma$ at which the creation of a vortex becomes energetically favorable (below, we set $r_0 \approx r_{core}$). In the regime $\rho e^2 d \ll 1$, we obtain $\gamma_c \approx \frac{1}{2ed}$. In general case, the energetic favorability of a vortex state depends on the integral $\int_{\mathbf{r}} \mathbf{B} \cdot \mathbf{H}$. An external field which is significantly stronger than that corresponding to $\gamma_c$ favors a higher phase winding number of the induced vortex structure, however, on the other hand, the vortex energy depends quadratically on $n$ [see Eq. (10)]. Therefore, a stronger field should normally produce a state with multiple one-dipolar-quantum vortices.

## III. KOSTERLITZ-THOULESS TRANSITION

In a planar $U(1)$-symmetric system, thermal fluctuations can excite finite-energy pairs of vortices with opposite windings. In model (6), the interaction between a vortex with $\Delta\theta = 2\pi$ located at $\mathbf{r}_1$ and an antivortex with $\Delta\theta = -2\pi$ located at $\mathbf{r}_2$ originates from two sources. The usual current-current interaction produces the attractive force,

$$\mathbf{F}_J = -2\pi\rho \left(1 - \frac{\rho e^2 d}{\rho e^2 d + 1}\right)^2 \frac{(\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^2}, \tag{11}$$

the other contribution comes from the $\mathbf{B}_{in}^2$ term,

$$\mathbf{F}_B = -2\pi\rho \frac{\rho e^2 d}{[\rho e^2 d + 1]^2} \frac{(\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^2}. \tag{12}$$

A system of these vortices and antivortices can be mapped onto a Coulomb gas which at a temperature $T_{\mathrm{KT}}$ undergoes a Kosterlitz-Thouless (KT) transition,

$$T_{\mathrm{KT}} = \frac{\pi}{2} \rho(T_{\mathrm{KT}}) \frac{1}{1 + \rho(T_{\mathrm{KT}}) e^2 d}. \tag{13}$$

Therefore, the temperature of the condensation transition in DEC will be suppressed compared to the value of the condensation temperature in a neutral system with similar density. While the suppression for realistic DECs is tiny, there is an interesting aspect in it because in contrast to the superfluid density jump $\rho / T_{\mathrm{KT}} = 2/\pi$ in regular superfluids, here one should define a "generalized superfluid density jump" which depends on a *nonuniversal* parameter: the layer separation. In the limit $\rho e^2 d \to \infty$, this temperature tends to zero reflecting the fact that in that limit, vortices do not have logarithmic interactions and thus there is no true KT transition.

As is well known, one of the ways to detect a KT transition and/or crossover in superconducting films is associated with a peculiar reaction to an applied current.[9] In a superconducting film, an external current results in the Lorentz forces acting on a vortex and antivortex in opposite ways, causing a pairbreaking effect. Free vortices create dissipation which is manifested in *IV* characteristics.[9] In DEC, vortices have magnetic field which can be viewed as that of magnetic charges and, correspondingly, they are sensitive to an applied external uniform in-plane magnetic field. Such a field, at finite temperature, should create a KT-specific modification of the zero-temperature response discussed in Ref. 4.

We note that because of small carrier density and layer separation in the presently available semiconductor bilayers,[7] a dipolar vortex would carry only about $10^{-7}$ flux quanta, which, though may be resolved with a modern superconducting quantum interference device, makes observation of these effects difficult. This raises the question if there could be strong-coupling dipolar superfluids in principle. Below, we show that the concept to dipolar superfluidity arises in a system principally different from DEC, without interlayer pairing problem (which limits dipolar coupling strength in DEC). This provides a possibility to have larger flux of a dipolar vortex and more pronounced phenomena associated with it. Moreover, there the concept of dipolar superfluidity allows generalization.

## IV. DIPOLAR SUPERFLUIDITY IN LAYERED SUPERCONDUCTORS

Consider a layered superconductor (LSC), i.e., a multiple superconducting layers separated by insulating layers (to effectively eliminate interlayer Josephson coupling), so that the layers are only coupled by the gauge field. This system has been extensively studied in the past.[10] In the hydrodynamic limit, its free energy density is

$$F = \sum_{i=1}^{N} \frac{1}{2} \delta(z_i) |(\nabla - ie\mathbf{A}(x,y,z_i)) \psi_i(x,y,z_i)|^2 + \frac{[\nabla \times \mathbf{A}(x,y,z)]^2}{2}, \tag{14}$$

where $\psi_i(x,y,z_i) = |\psi_i(x,y,z_i)| e^{i\theta_i(x,y,z_i)}$, $z_i + d_i = z_{i+1}$. This model indeed does not feature dipolar superfluidity of the DEC type. Here, the main distinction is the reaction to the external field which is screened because of the Meissner effect [the effective screening length in a planar superconductor is $\lambda^2$/(layer thickness)]. However, there are situations where the superconductivity can be eliminated in this system by topological defects. That is, if to apply an external field along the $z$ direction, one can produce a lattice of vortex lines with phase winding in each layer $[\Delta\theta_1(z_1)=2\pi,\ldots,\Delta\theta_N(z_N)=2\pi]$.[10] Enclosed magnetic flux gives such a vortex line a finite tension. At elevated temperatures, the lattice of these vortices melts but importantly normally there is a range of parameters (which depends on the strength of the applied field and temperature) where the vortex lines forming a liquid retain the tension.[10–12] If the vortex lattice is not pinned or if one has a tensionfull vortex liquid, the charge transfer in superconducting layers is dissipative. However, in such situations, a system nonetheless retains broken symmetries associated with the phase differences between the order parameters and, correspondingly, dissipationless countercurrents;[13] in this particular case, a broken symmetry is retained in the phase difference between the layers.[13] Physically, the situation which occurs is as follows: consider the $N=2$, $|\psi_1|=|\psi_2|=|\psi|$ case. Currents in individual layers move unpinned vortices which produce dissipation. However, as long as a vortex line threading the system has a finite tension, equal countercurrents in different layers deform but not move a vortex line and therefore do not create dissipation. For small layer separation $d$, the dissipationless counterflows can approximately be described by extracting phase difference terms.[13] Then, the part of model (14) which retains a broken symmetry is

$$F_d \approx \frac{1}{4}|\psi|^2\{\nabla[\theta_1(x,y,z_1)-\theta_2(x,y,z_2)] - e[\mathbf{A}(x,y,z_1)-\mathbf{A}(x,y,z_2)]\}^2 + \frac{d}{2}\mathbf{B}_{in}^2. \quad (15)$$

The vortices in this system are related to dipolar vortices in DEC. The simplest vortices with a topological charge in the phase difference are $(\Delta\theta_1=\pm 2\pi,\ \Delta\theta_2=0)$ and $(\Delta\theta_1=0,\ \Delta\theta_2=\pm 2\pi)$. We denote them by a pair of integers $(\pm 1,0)$ and $(0,\pm 1)$. For a vortex $(\pm 1,0)$, the currents in two layers are $\mathbf{j}_1=e|\psi|^2\nabla\theta_1-e^2|\psi|^2\mathbf{A}(x,y,z_1)$ and $\mathbf{j}_2=-e^2|\psi|^2\mathbf{A}(x,y,z_2)$. Like in a DEC, in the case of a finite $d$, the vortex features in-plane radial magnetic field $[\hat{R}\mathbf{B}_{in}]\approx\partial_z\mathbf{A}(\mathbf{r})$. These vortices can be induced by an external in-plane magnetic field, like in a DEC. However, there are indeed also principal differences. LSC is a system with more degrees of freedom and the above considerations apply only to the state when superconductivity in individual layers is removed, e.g., by a molten lattice of (1,1) vortices, or thermally excited $(\pm 1,\pm 1)$ vortices. We stress that in this system, there is no interlayer pairing, but counterflow is the only surviving type of dissipationless charge transfer. Note also that the molten lattice of (1,1) vortices does not automatically preclude a formation of an ordered state of $(\pm 1,0)$ and $(0,\pm 1)$ vortices because a vortex (1,1) does not have a topological charge in the phase difference sector $\nabla(\theta_1-\theta_2)$, and their disordered states cannot eliminate corresponding phase stiffness. The density of (1,1) vortices and correspondingly the temperature of their lattice melting are controlled by the strength of the magnetic field in the $z$ direction, while the density of $(\pm 1,0)$ and $(0,\pm 1)$ vortices is controlled by the in-plane magnetic field. The system therefore possesses a control parameter which allows for the ordered structures of $(\pm 1,0)$ and $(0,\pm 1)$ vortices to coexist with a liquid of (1,1) vortices.

The dipolar superfluidity in LSC should have a number of detectable physical consequences. Namely, the LSC in the vortex liquid state should have a dipolar superfluid response, analogous to that discussed in great detail in Ref. 4. Also, the system in the vortex liquid state should possess aspects of planar electrodynamics with effective magnetic charges discussed above in connection with DEC which may be observable in the temperature dependency of the dipolar response. In the both cases of DEC and LSC, the dipolar superfluidity will be destroyed by interlayer tunneling, which amounts to explicit symmetry breakdown, but for very small interlayer tunneling, some of its signatures will, in some cases, remain.[3]

## V. MULTIFLAVOR DIPOLAR SUPERFLUIDITY

The general case of $N$ layers, especially with the variable interlayer distances and the condensate densities, has much richer structure than DEC because the dipolar superfluid modes are multiple and coupled. That is, for $N$ layers, one can have multiple combinations of counterflows in different layers which will not move a tensionfull vortex line. For example, consider unpinned vortex lattice or tensionfull vortex liquid in $N=3$ case. Then, there are combinations of current in one of the layers accompanied by (weaker) countercurrents in the other two layers which may deform but will not move a tensionfull vortex line. Therefore, for $N>2$, dipolar modes can no longer be associated with counterflows in just two layers, but one has to consider all possible combinations of phase differences (i.e., all possible realizations of counterflows in different layers) to describe dipolar modes. In the limit of very small $d$, the kinetic terms for countercurrents approximately can be expressed as

$$F_d^{CF} \approx \sum_{i,j=1}^{N}\frac{|\psi|^2}{4N}\{\nabla[\theta_i(x,y,z_i)-\theta_j(x,y,z_j)] - e[\mathbf{A}(x,y,z_i)-\mathbf{A}(x,y,z_j)]\}^2. \quad (16)$$

Observe that the dipolar modes are multiple and not independent, which is very different from $N=2$ case and cannot be directly mapped on DEC. Rather, it generalizes the dipolar superfluidity concept to the "multiflavor" case.

## VI. CONCLUSION

We have considered the possible physical effects dictated by the symmetry and dynamics of a gauge field in dipolar condensates. For DEC, we started by constructing an effective model, based on symmetry and energy scales of the problem. In this framework, we described topological defects which emit a nonuniversally quantized radial magnetic flux. We pointed out that because of the existence of well separated energy scales in the regime, when interlayer distance is smaller than other length scales in the problem, the system possesses effective planar electrodynamics with magnetic charges (planar analog of magnetic monopoles) arising from topological excitations. Therefore, at finite temperature, an applied in-plane magnetic field should have a vortex-antivortex pairbreaking effect. Experimentally, it may manifest itself through a counterpart of the Halperin-Nelson response which may be observable in systems with sufficiently strong dipolar coupling. In the second part of the paper, we map DEC onto the tensionful vortex liquid state in layered superconductors. In that system, we point out the emergence of a dipolar superfluidity which has a different origin because there is no interlayer pairing and carriers in layers have the same sign of electric charge. There the analog of the dipolar superfluidity arises as a consequence of the fact that an unpinned or molten lattice of composite vortices (or thermally excited unpaired composite vortices) makes currents in individual layers dissipative while the countercurrents in different layers remain dissipationless. In these systems, there is no need for interlayer pairing and carrier density is higher, so the effective dipolar coupling may be relatively large. Besides that, the dipolar response can be used as an experimental tool to study vortex liquids in LSC, e.g., to unequivocally distinguish a tensionfull vortex liquid state from tensionless vortex tangle. Finally, we have shown that a vortex liquid state in a *N*-layer superconductor represents a generalization of the dipolar superfluidity concept to the multiflavor case where there are multiple and topologically coupled dipolar modes.

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